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INTRODUCTION TO DIFFERENTIATION.

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In the identity

$$\frac{r^n-1}{r-1}=r^{n-1}+r^{n-2}+\dots+r+1, \tag{1}$$

since r may have any value, let $r=\frac{a^{\frac{1}{m}}}{b^{\frac{1}{m}}}$; then by substituting this value for r in (1), multiplying both members by $b^{\frac{n-1}{m}}$, and simplifying, we obtain:

$$\frac{a^{\frac{n}{m}}-b^{\frac{n}{m}}}{a^{\frac{1}{m}}-b^{\frac{1}{m}}}=a^{\frac{n-1}{m}}+a^{\frac{n-2}{m}}b^{\frac{1}{m}}+\dots+a^{\frac{1}{m}}b^{\frac{n-2}{m}}+b^{\frac{n-1}{m}}. \tag{2}$$

A result from which, since m may have any value (± 1 included), can be deduced all the theorems regarding the divisibility of $x^n \pm y^n$.

Dividing both members of (2) by the factor that renders $a^{\frac{1}{m}}-b^{\frac{1}{m}}$ rational, we obtain

$$\frac{a^{\frac{n}{m}}-b^{\frac{n}{m}}}{a-b}=\frac{a^{\frac{n-1}{m}}+a^{\frac{n-2}{m}}b^{\frac{1}{m}}+\dots+a^{\frac{1}{m}}b^{\frac{n-2}{m}}+b^{\frac{n-1}{m}}}{a^{\frac{m-1}{m}}+a^{\frac{m-2}{m}}b^{\frac{1}{m}}+\dots+a^{\frac{1}{m}}b^{\frac{m-2}{m}}+b^{\frac{m-1}{m}}}, \tag{3}$$

since, by (2), $a-b=(a^{\frac{1}{m}}-b^{\frac{1}{m}})(a^{\frac{m-1}{m}}+\dots+b^{\frac{m-1}{m}})$.

In (3), if we suppose $b=a$, then, since in the numerator of the second member we shall have n terms each $=a^{\frac{n-1}{m}}$, and, in the denominator, m terms each $=a^{\frac{m-1}{m}}$ we obtain

$$\left[\frac{a^{\frac{n}{m}}-b^{\frac{n}{m}}}{a-b}\right]_{a=b}=\frac{0}{0}=\frac{na^{\frac{n-1}{m}}}{ma^{\frac{m-1}{m}}}=\frac{n}{m}a^{\frac{n}{m}-1}, \tag{4}$$

the first member assuming the indeterminate form from the presence, in the numerator and denominator, of the factor $a^{\frac{1}{m}}-b^{\frac{1}{m}}$ which becomes zero by the supposition. Hence, since m may have any value, the formula

$$\left[\frac{x^n-y^n}{x-y}\right]_{x=y}=nx^{n-1} \tag{5}$$

holds true for every value of n.

Let $u = x^n$, and $u' = (x+h)^n$, then the ratio of the increase of x^n to that of x is expressed, for all values of h, by

$$\frac{u'-u}{h} = \frac{(x+h)^n - x^n}{(x+h) \; - x}. \tag{6}$$

If now we suppose $x = x+h$, i. e., $h = 0$, then by (5), we obtain

$$\frac{0}{0} = n x^{n-1}, \tag{7}$$

the first member assuming the indeterminate form, as in (4), from the presence of a factor that has become zero in both its terms. Supposing this common factor to be removed by division, and denoting the result by

$\frac{dx^n}{dx}$, we have the result $\frac{dx^n}{dx} = n x^{n-1}$

(where n may have any value), which may be thus defined: *If in the function* x^n, x *be supposed to increase continuously, then, when* x *reaches any assigned value* x' *the rate at which* x^n *is increasing, as compared with that of* x, *is* nx'^{n-1}, *that is, if we denote an infinitesimal change in* x *by* dx, *the corresponding change in* x^n *is* $nx^{n-1}dx$. *This last expression is called &c.*

The difficulty which here besets us arises from our inability to conceive an absolute infinitesimal in time or space, and is really identical with that we meet in questions regarding motions. Thus, for example, the velocity attained by a body falling under certain conditions, is expressed by $v = gt$. Now before the end of the t^{th} unit of time $v < gt$, and after the end of the t^{th} unit of time $v > gt$, nor are we able to conceive any portion of time, however minute, during which the body moves with the velocity gt. Yet, as the velocity increases continuously from less than to greater than gt, we are forced to the conclusion that for some infinitesimal portion of time $v = gt$. By performing the division indicated in the second member of (6) we obtain:

$$\frac{u'-u}{h} = (x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}; \tag{8}$$

from which we see that $[(u - u') \div h] > nx^{n-1}$, if h, however conceivably small, be positive, and $[(u - u') \div h] < nx^{n-1}$, if h be negative. Hence, if we conceive x^n to vary continuously we are forced to conclude that, when x reaches any assigned value x', then, corresponding to any infinitesimal change in x' there is a simultaneous change in x^n that is nx'^{n-1} times as great, which is expressed by the formula

$$\frac{dx^n}{dx} = n x^{n-1}.$$